Change of Variables

1. Spherical Jacobian

At the beginning of class, we saw that the 2D Jacobian for the transform from Cartesian to polar coordinates is r. Compute a 3D Jacobian for the transform to spherical coordinates. Is this what you expect?

2. uv Substitution.

Consider the integral $\int_0^1 \int_{y/2}^{y/2+2} (2x - y)dxdy$. Let $u = 2x - y$ and let $v = y$. Transform and solve.

Change of Variables continued on next page...

3. Exam Style Change of Variables.

Consider a trapezoidal metal plate in the $x-y$ plane with vertices $(1,\ 0), (2,\ 0), (0,\ 1)$ and $(0,\ 2)$. The density of the plate (mass per unit area) is given by $\cos\left(\frac{y-x}{y+x}\right)$.

(a) Write out the integral required to determine the mass of the plate in the order $dydx$. Also, sketch the region of integration R in the $x-y$ plane. Be sure to label all axes, boundaries, etc... on your sketch.

(b) Now use the transformation $u = y + x$ and $v = y - x$. Solve for x and y in terms of u and v. Be sure to check this result!

(c) Map the region R into its corresponding region S in the $u-v$ plane and make a sketch of it. Be sure to label all axes, boundaries, etc. on your sketch.

(d) Calculate the Jacobian, $J(u, v)$.

(e) Rewrite your integral in terms of u and v. Select an order of integration that will result in one integral. Be sure to include the limits of integration.

(f) Finally, evaluate your integral from part (e) to determine the mass of the plate.