Worksheet 2

10.3 - The Dot Product

1. Find the dot product between $\mathbf{a} = (a_1,\ a_2,\ a_3,\ \ldots,\ a_n)$ and $\mathbf{b} = (b_1,\ b_2,\ b_3,\ \ldots,\ b_n)$ in $\mathbb{R}^n$.

2. What is the angle θ between $\mathbf{a}$ and $\mathbf{b}$ in $\mathbb{R}^n$. (Hint: Use the other expression for the dot product.)

3. Using your previous answer. When is the dot product $\mathbf{a} \cdot \mathbf{b} = 0$?

4. Recall that the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\text{comp}_{\mathbf{a}}\mathbf{b} = (\mathbf{a} \cdot \mathbf{b})/|\mathbf{a}|$. This is the magnitude of the *component* of b along a. What is the vector projection of $\mathbf{b}$ onto $\mathbf{a}$? Why does this make sense?

5. Suppose you have a force given by the vector $\mathbf{F} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ and that this force moves a particle from $P = (-2,\ 3,\ 2)$ to $R = (1,\ 5,\ -4)$. Use the dot product to find the work done.

10.4 - The Cross Product

1. What is the formula for the cross product $\mathbf{a} \times \mathbf{b}$ in $\mathbb{R}^3$?

2. Find the angle θ between $\mathbf{a}$ and $\mathbf{b}$ in terms of the cross product.

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3. If $\mathbf{a} = (2,\ -1,\ 1)$, $\mathbf{b} = (1,\ 1,\ -1)$, and $\mathbf{c} = (3,\ -\frac{3}{2},\ \frac{3}{2})$, use the cross product and the dot product to determine which vectors are orthogonal and which are parallel.

4. Suppose you have a parallelogram formed by $\mathbf{a} = (2,\ -1,\ -3)$ and $\mathbf{b} = (1,\ 1,\ 1)$. What is the area?

5. Suppose now that you have a 3-dimensional parallelepiped formed by $\mathbf{a} = (2,\ -1,\ -3)$, $\mathbf{b} = (1,\ 1,\ 1)$, and $\mathbf{c} = (7,\ -\frac{1}{2},\ 4)$. What is the volume of the parallelepiped formed by these vectors?

6. Suppose you tighten a bolt by applying a 50-N force to a 0.2-m wrench. Suppose the force is applied in a direction that is perpindicular to the axis of the wrench. What is the torque (magnitude and direction) of the right threaded bolt?

10.5 - Equations of Lines and Planes

1. Suppose you have a point $P_0 = (-7,\ 5,\ 0)$ and a point $P_1 = (4,\ -6,\ -\frac{7}{2})$.
   (a) Find a vector equation for the line L that passes through these two points.
   (b) Find a parametric equation for the same line.
   (c) Find a different point P_2 on the line.

2. Find a vector equation of a plane P which is parallel to line L from the previous problem and which is orthogonal to $\mathbf{n} = -\mathbf{i} - \mathbf{j}$.

3. Give a scalar equation for this plane P.