Worksheet 5

11.3 - Partial Derivatives

1. *(Fill in the blank)* **CLAIRAUT'S THEOREM:** Suppose f is defined on a disk D that contains the point (a, b). If the functions f_{xy} and f_{yx} are both continuous on D, then ______________________________.

2. Verify that the conclusion of Clairaut's Theorem holds, that is, verify that $u_{xy} = u_{yx}$ for $u = x^5y^4 - 3x^2y^3 + 2x^2$.

3. Show that $u = (x - at)^6 + (x + at)^6$ is a solution of the partial differential equation $u_{tt} = a^2u_{xx}$. This equation is known as the wave equation.

11.4 - Tangent Planes and Linear Approximations

1. *(Fill in the blank)* Suppose f has continuous partial derivatives. An equation of the tangent plane to the surface $z = f(x, y)$ at the point $P(x_0, y_0, z_0)$ is ____.

2. *(Fill in the blank).* If the partial derivatives f_x and f_y exist near (a, b) and are continuous at (a, b), then ____

3. *(Fill in the blank).* For a differentiable function of two variables, $z = f(x, y)$, we define the ____________________ dx and dy to be independent variables; that is they can be given any values. Then the differential, dz, also called the ____, is defined by
$dx =$ ____.

4. Find the equation of the tangent plane to $z = e^{x^2-y^2}$ at $(1, -1, 1)$.

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5. Find the linearization of $f(x,y) = \sin x + \cos y$ at (π, π) and use it to approximate $f(3.1, 2.9)$.

11.5 - Chain Rule

1. *(Fill in the blank)* **CHAIN RULE (CASE 1):** Suppose $z = f(x,y)$ is a differentiable functions of x and y where $x = g(t)$ and $y = h(t)$ are both differentiable functions of t. Then z is a differentiable function of t and $\frac{dz}{dt} =$ ________________________________.

2. *(Fill in the blank)* **CHAIN RULE (CASE 2):** Suppose $z = f(x,y)$ is a differentiable functions of x and y where $x = g(s,t)$ and $y = h(s,t)$ are both differentiable functions of s and t. Then $\frac{dz}{ds} =$ ______________________ and $\frac{dz}{dt} =$ ______________________.

3. At what rate is the volume of a box changing if its length is 8 ft and increasing at 3 ft/s, its width is 6 ft and increasing at 2ft/s, and its height is 4ft and increasing at 1ft/s?