Lagrange Multipliers

1. Lagrange multipliers

Recall that last week, we considered finding the local max/min of the function $f(\vec{x})$ subject to the constraint $g(\vec{x}) = c$. Now let's consider the same problem with Lagrange multipliers. One interpretation is that the Lagrange multiplier represents a penalty. Then the problem minimize $f(\vec{x})$ such that $g(\vec{x}) = c$ can be thought of as minimizing $f(\vec{x}) + \lambda|c - g(\vec{x})|$. By setting the gradient of this system equal to zero, we find that critical points satisfy

$$\begin{cases} \vec{\nabla} f = \lambda \vec{\nabla} g, \\ g(\vec{x}) = \vec{c}. \end{cases}$$

Use this to minimize $x^2 + y^2 + z^2$ subject to $x + 2y + z = 6$.

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2. Maximizing volume of a can.

Suppose you work for Duff Beer and you have been tasked with maximizing the volume of beer cans subject to the constraint that the manufacturing budget is $ 1 per can. Furthermore, the company's market analysts have informed you that manufacturing costs are $3¢/cm.^2$ for the sides and $6¢/cm.^2$ for the top and bottom. Propose a solution to company executives that maximizes the volume of the can subject to the constraint.